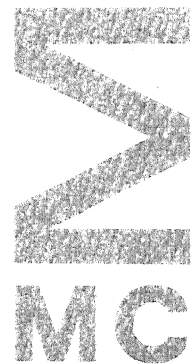


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AFDELING ZUIVERE WISKUNDE
(DEPARTMENT OF PURE MATHEMATICS)

ZN 87/78

DECEMBER

J. VAN DE LUNE

A NOTE ON A PROBLEM OF ERDÖS

amsterdam

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A note on a problem of Erdős

by

J. van de Lune

ABSTRACT

This note contains a method for finding natural numbers n such that 2^n starts with the same ordered sequence of digits as n does. Six numbers of this kind are presented.

Recently my attention was drawn to the following observation made by P. Erdős: $2^6 = 64$ and $2^{10} = 1024$. Here we have two examples of the phenomenon that the number 2^n (written, as usual, in the scale of ten) starts with the same ordered sequence of digits as the natural number n itself.

Let us call a positive integer having this property an Erdős - number.

We use the following notation: $[x]$ denotes the integral part of the real number x ; $\{x\} := x - [x]$ denotes the fractional part of x and the base ten logarithm of $x \in \mathbb{R}^+$ will be written as $\text{LOG } x$.

From the first principles of our decimal number system it follows that the number of digits $d(n)$ of a natural number n is given by

$$d(n) := [\text{LOG } n] + 1$$

and that the first digit $f(n)$ of $n \in \mathbb{N}$ is given by

$$f(n) := \left[\frac{n}{10^{d(n)-1}} \right] = [10^{\text{LOG } n - [\text{LOG } n]}].$$

Some elementary decimal point manipulation shows that $n \in \mathbb{N}$ is an Erdős - number if and only if

$$n = \left[\frac{2^n}{10^{d(2^n)-1}} * 10^{d(n)-1} \right] = [10^{\text{LOG } 2^n - [\text{LOG } 2^n] + [\text{LOG } n]}],$$

which subsequently, is equivalent to

$$n \leq 10^{n \text{ LOG } 2 - [n \text{ LOG } 2] + [\text{LOG } n]} < n+1,$$

$$\text{LOG } n \leq n \text{ LOG } 2 - [n \text{ LOG } 2] + [\text{LOG } n] < \text{LOG}(n+1),$$

or, finally

$$0 \leq \{n \text{ LOG } 2\} - \{\text{LOG } n\} < \text{LOG}(n+1) - \text{LOG } n,$$

a formula which is very convenient for finding large Erdős - numbers.

On a small programmable pocket calculator (an HP-65 in our case) we ran the following program:

<u>LBL A</u>	RCL 1
STO 1	1
f LOG	+
STO 2	STO 1
2	f LOG
f LOG	STO 2
STO 3	+
<u>LBL B</u>	$g \times \leq y$
RCL 1	GTO B
RCL 3	RCL 1
*	1
f^{-1} INT	-
RCL 2	R/S
f^{-1} INT	LBL C
-	RCL 1
0	1
$g \times > y$	+
GTO C	STO 1
$g \downarrow$	f LOG
RCL 2	STO 2
CHS	GTO B

Before running this program by pressing key A, one has to key in the number n one wants to start with. We simply started with $n = 1$. At R/S the display showed successively $n = 6$, $n = 10$ and after quite a while, $n = 1542$. After these results the above program was run during an entire weekend without any further result.

After this, the above program was translated into FORTRAN and implemented on a CDC Cyber 73/173. In a few seconds we found the following Erdős - numbers: $n = 77075$, $n = 113939$ and $n = 1122772$.

Finally the same program was run (with double precision arithmetic) for several hours with the result that the previously found Erdős - numbers are the only ones in the range $n \leq 5 * 10^7$.

Conclusion: In the range $n \leq 5 * 10^7$ there are six Erdős - numbers, to wit:

n = 6
n = 10
n = 1542
n = 77075
n = 113939
n = 1122772.