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AFDELING ZUIVERE WISKUNDE
(DEPARTMENT OF PURE MATHEMATICS)

ZN 87/78 DECEMBER

J. VAN DE LUNE

A NOTE ON A PROBLEM OF ERDÖS

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A NOTE ON A PROBLEM OF ERDÖS

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A note on a problem of Erdös

by

J. van de Lune

ABSTRACT

This note contains a method for finding natural numbers n such that 2^n starts with the same ordered sequence of digits as n does. Six numbers of this kind are presented.

Recently my attention was drawn to the following observation made by P. Erdős: $2^6 = 64$ and $2^{10} = 1024$. Here we have two examples of the phenomenon that the number 2^n (written, as usual, in the scale of ten) starts with the same ordered sequence of digits as the natural number n itself.

Let us call a positive integer having this property an Erdős - number.

We use the following notation: [x] denotes the integral part of the real number x; $\{x\} := x - [x]$ denotes the fractional part of x and the base ten logarithm of $x \in \mathbb{R}^+$ will be written as LOG x.

From the first principles of our decimal number system it follows that the number of digits d(n) of a natural number n is given by

$$d(n) := [\mathrm{LOG}\ n] + 1$$

and that the first digit f(n) of $n \in \mathbb{N}$ is given by

$$f(n) := \left[\frac{n}{10^{d(n)-1}}\right] = \left[10^{\mathrm{LOG}\ n - [\mathrm{LOG}\ n]}\right].$$

Some elementary decimal point manipulation shows that $n \in \mathbb{N}$ is an Erdős - number if and only if

$$n = \left[\frac{2^n}{10^{d(2^n)-1}} * 10^{d(n)-1}\right] = \left[10^{\mathrm{LOG}\ 2^n - [\mathrm{LOG}\ 2^n] + [\mathrm{LOG}\ n]}\right],$$

which subsequently, is equivalent to

$$n \le 10^{n\ \mathrm{LOG}\ 2 - [n\ \mathrm{LOG}\ 2] + [\mathrm{LOG}\ n]} < n+1,$$

$$\mathrm{LOG}\ n \le n\ \mathrm{LOG}\ 2 - [n\ \mathrm{LOG}\ 2] + [\mathrm{LOG}\ n] < \mathrm{LOG}(n+1),$$

or, finally

$$0 \le \{n\ \mathrm{LOG}\ 2\} - \{\mathrm{LOG}\ n\} < \mathrm{LOG}(n+1) - \mathrm{LOG}\ n,$$

a formula which is very convenient for finding large Erdős - numbers.

On a small programmable pocket calculator (an HP-65 in our case) we ran the following program:

```
LBL A
STO 1
f LOG
STO 2
  2
f LOG
STO 3
LBL B
RCL 1
RCL 3
  *
f^-1 INT
RCL 2
f^-1 INT
  -
  0
g x > y
GTO C
  g ↓
RCL 2
CHS
RCL 1
  1
  +
STO 1
f LOG
STO 2
  +
g x ≤ y
GTO B
RCL 1
  1
  -
 R/S
LBL C
RCL 1
  1
  +
STO 1
f LOG
STO 2
GTO B
```

Before running this program by pressing key A, one has to key in the number n one wants to start with. We simply started with $n = 1$. At R/S the display showed successively $n = 6$, $n = 10$ and after quite a while, $n = 1542$. After these results the above program was run during an entire weekend without any further result.

After this, the above program was translated into FORTRAN and implemented on a CDC Cyber 73/173. In a few seconds we found the following Erdõs - numbers: $n = 77075$, $n = 113939$ and $n = 1122772$.

Finally the same program was run (with double precision arithmetic) for several hours with the result that the previously found Erdõs - numbers are the only ones in the range $n \leq 5 * 10^7$.

Conclusion: In the range $n \leq 5 * 10^7$ there are six Erdõs - numbers, to wit:

n = 6

n = 10

n = 1542

n = 77075

n = 113939

n = 1122772.